AXIAL VIBRATION CHARACTERISTICS OF METAL-FLEXING COUPLINGS

by

Fred K. Landon

Manager, Coupling Division

Rexnord Inc.

Warren, Pennsylvania

and

Louis F. Counter

Senior Research Manager, Corporate R&D Group

Rexnord Inc.

Milwaukee, Wisconsin

Fred K. Landon, engineering manager in the Coupling Division of Rexnord Inc., graduated from the University of Maryland in 1959 with a B.S. Degree in Aeronautical Engineering. He also did graduate study at Rensselaer Polytechnic Institute and Johns Hopkins University. Mr. Landon is an author of papers delivered at the Cooling Tower Institute and National Conference on Power Transmission. He is a contributing author to McGraw-Hill's "Pump Handbook" and is the holder of a U.S. patent on an elastomer-type flexible coupling. He is a registered professional engineer in the State of Maryland.

Louis F. Counter received a B.S.M.E. degree in 1964 and a M.S.M.E. degree in 1966 from Iowa State University. He has been with Rexnord Inc. for ten years. As a senior research manager in the Corporate R&D Group, he is responsible for product testing and data development on many product lines. His special areas of expertise include experimental mechanics and product performance testing.

Mr. Counter has presented papers to the SESA, CEMA, and the Fatigue Design and Evaluation Committee of SAE.

ABSTRACT

Increased usage of metal-flexing couplings on high-speed rotating machinery has created awareness of the axial resonant vibration characteristics of this type of coupling. A testing program has been conducted at Rexnord Research and Development Laboratory; this program has been directed toward a quantified study of the phenomenon, as well as correlation of measured and calculated values, so as to avoid operational problems during the design phase of the machinery train. In addition, techniques are presented which permit the coupling designer to predictably modify the coupling and thereby make in-place retrofits should an axial resonance condition occur in the field.

INTRODUCTION

The first all-metal coupling to utilize the flexibility of steel components was introduced in 1915 by the Thomas Flexible Coupling Company. Over the years, this type of flexible coupling has been applied to virtually every type of mechanical drive application in existence. Approximately 20 years ago, this style of coupling was first applied to so-called "high-speed" rotating machinery.

The definition of a "high-speed" coupling application is difficult to establish. In general terms, a high-speed coupling is used wherever it is necessary to maximize torque-carrying capacity (generally, though not always, at the expense of misalignment capability) and to minimize the coupling-induced overhung weight and moment on connected equipment. Quantitatively, it is possible to relate the term to the peripheral velocity at the coupling flange outside diameter (see Figure 1). It is generally agreed that a coupling operating at a

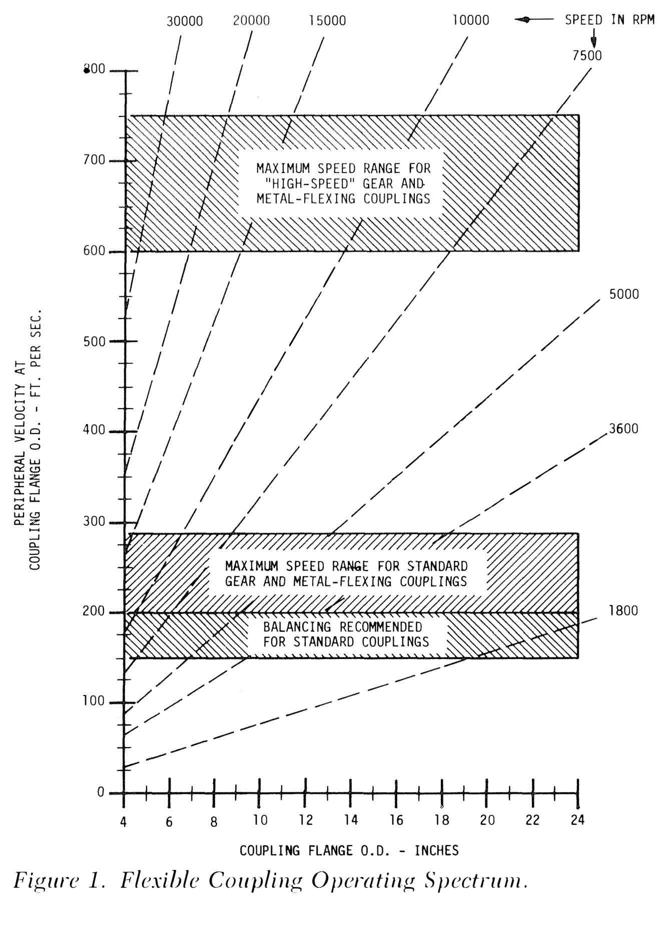

Figure 1. Flexible Coupling Operating Spectrum.

peripheral speed of 250 ft./sec. or greater — that is, a six-inch diameter coupling operating at about 10,000 RPM — falls into the category of high-speed operation. Note, however, that coupling manufacturers generally recommend dynamic balancing at about one-half this speed.

Rapid advancements in the state of the art of high-speed turbomachinery occurred in the 1950's and 1960's as process technology demands on rotating equipment increased. Specific output of pumps and compressors rose along with a corresponding decrease in the size of the rotating elements. Concurrently, users began making increasingly stronger demands on equipment builders to produce machinery which could operate for extended periods of time without shutdown.

A consequence of these developments was that the flexible coupling became an ever-increasingly important (and critical) component in the drive train. In some cases, it was physically impossible to fit a thrust bearing in a housing which would adequately absorb the thrust transmitted by a locked gear coupling. Likewise, in many installations, lubricant filtration technology was inadequate to prevent sludge accumulation in the teeth of a gear coupling. It was about that time that machinery designers and users began taking a serious look at all-metal, non-lubricated, material-flexing couplings — first, as "problem solvers" for difficult applications, but more recently as standard equipment for high-speed compressor and pump drive applications.

At one time, these metal-flexing designs were thought of as too heavy and incapable of achieving and maintaining the dynamic balance accuracy required on high-speed equipment. However, technology has improved, as well as design and manufacturing expertise, to the point where this type of coupling is now suitable for most applications employing high-speed couplings. Several different designs are available, but all have in common the basic concept of utilizing the flexibility of a metallic component (usually steel) to accommodate misalignment between shafts.

All contemporary metal-flexing coupling designs have one thing in common: the coupling assembly (which consists of two fixed hubs, two flexing elements, and a floating spacer piece) is a basic spring-mass system. As such, it has the potential to vibrate when excited at its resonant frequency. It is the purpose of this paper to examine the phenomenon from both an analytical and experimental standpoint and to offer techniques for the avoidance of resonant excitations both in the equipment design stage and on a retrofit or "field-fix" basis.

STATEMENT OF THE PROBLEM

A metal-flexing coupling typically consists of the following components (see Figure 2):

1. Two hubs, rigidly attached by interference fit or flange bolting to the driving and driven shafts of the connected equipment;
2. Two flexing elements (laminated or single-piece), one attached to each hub, which compensate for misalignment;
3. A distance piece or spacer (usually tubular, occasionally solid) which spans the gap between shafts and is attached at each end to the flexing elements.

Consider the functional requirements and characteristics of the flexing elements. They must transmit rated torque as well as any system overloads without buckling or permanently deforming; in other words, they must possess torsional rigidity. However, under conditions of parallel, angular, and axial misalignment, the flexing element must have sufficient flexibility

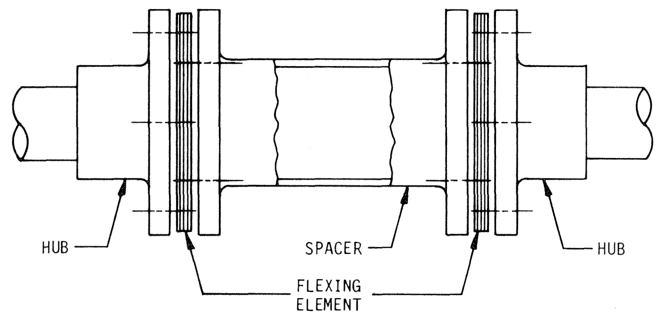

Figure 2. Typical Metal-Flexing Coupling.

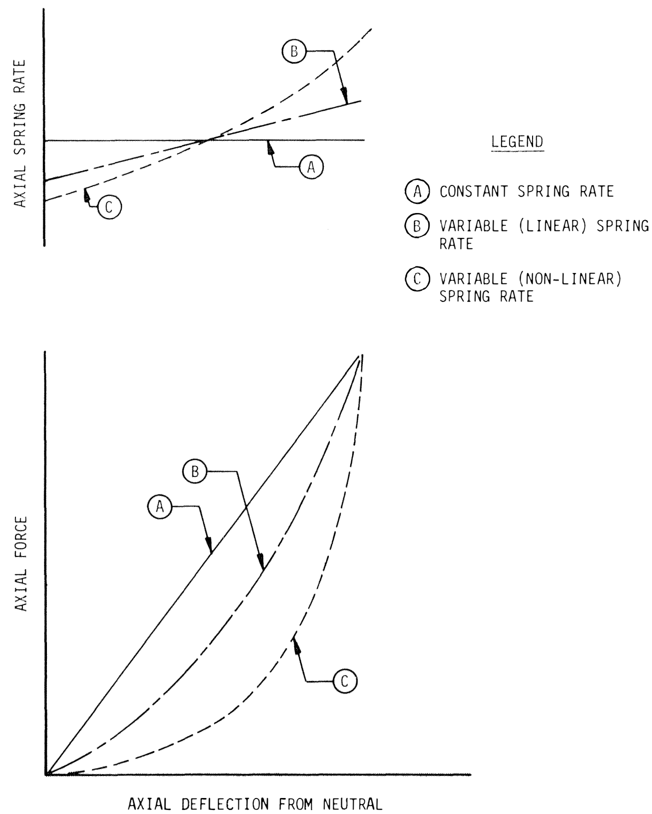

Figure 3. Coupling Spring Rate Characteristics.

to accommodate these conditions without imposing excessive forces and moments on equipment shafts and bearings. Both of the above requirements must be met while maintaining stress levels which are safely within the fatigue limit of the flexing material.

Since this type coupling relies on elastic material deformation to achieve flexibility, it exhibits predictable and consistent spring rates under each degree of freedom. Considering specifically the axial direction of motion, the spring rate characteristics may vary substantially, depending on the coupling design. Referring to Figure 3, the coupling may have: (A) - a constant, (B) - linearly-variable, or (C) - non-linearly variable spring rate.

Metal-flexing couplings have been known to occasionally exhibit large-amplitude vibrations in the axial direction when excited at the natural frequency of the coupling. The possibility of such an occurrence becomes obvious if the coupling is visualized as a spring-mass system as shown on Figure 4. The following sections of this paper will analyze and experimentally verify the behavior of such a system when subjected to external excitation.

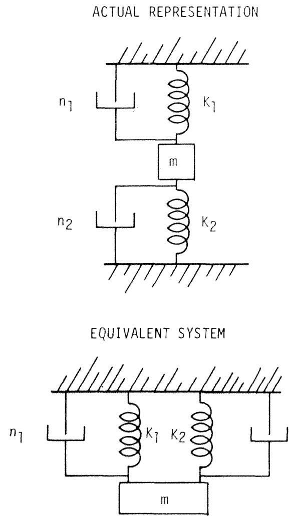

Figure 4. Mechanical Analogy of Metal-Flexing Coupling.

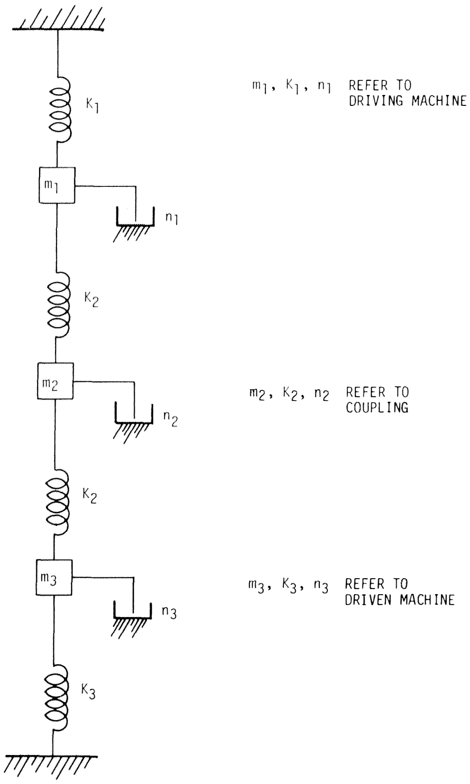

Figure 5. Mass-Elastic Diagram of Two-Mass Drive System.

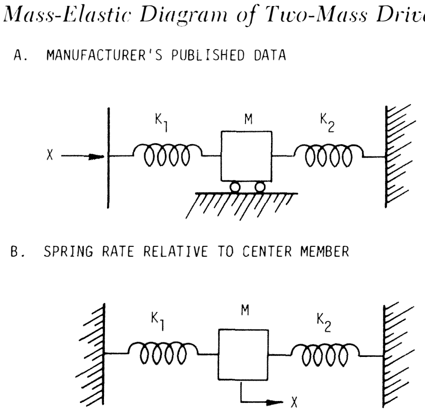

Figure 6. Interpretation of Coupling Spring Rates.

ANALYTICAL SOLUTION

Referring again to Figure 4, a metal-flexing coupling can be represented by a mass, m, supported between two springs having stiffness of K_1 and K_2. Assuming that some amount of damping is present in the spring members, this can be represented by n_1 and n_2, respectively. Since it is highly unusual to construct a flexible coupling with flexing elements of different characteristics at each end, it is reasonable to assume that $K_1 = K_2 = K$, and $n_1 = n_2 = n$.

The system of Figure 4 may be further simplified into a basic spring-mass system of the type shown in Figure 5, which shows the coupling in relation to the driving and driven masses. Here, it should be pointed out to exercise caution when applying a coupling manufacturer's spring rate data to the analysis of an axial resonance condition. Published data generally describes the coupling stiffness as if a force were applied at one hub; the resistance of the coupling to this motion is determined by the spring rate data (Figure 6a). Here, the effective stiffness of the coupling is *one-half* that of an individual spring (or flexing element). When analyzing the vibration of the center member, the system is treated as two springs in parallel (Figure 6b), and the effective stiffness is *twice* that of an individual spring. Thus, when using manufacturers' data, the published spring rate must be multiplied by a factor of four to analyze axial resonant vibrations.

The amount of damping present in a metal-flexing coupling is thought to be relatively small, although it is known to be greater for the laminated-disc type construction than for a coupling consisting of a single-piece membrane. The reason for the greater damping in the laminated disc configuration is, that under conditions of axial movement, a microscopic amount of motion takes place between adjacent lamina, as shown in Figure 7. Since the element is clamped together under a bolt preload, P, there is a frictional force, f, which resists sliding. The frictional force provides coulomb damping (1), and the component fx thereby damps the vibratory motion in the axial direction.

The equation describing the behavior of such a system is given (2) as follows:

$$\frac{A}{\frac{Fo}{K}} = \frac{1}{\sqrt{\left[1-\left(\frac{w}{p}\right)^2\right]^2 + \left[2\left(\frac{n}{n_c}\right)\left(\frac{w}{p}\right)\right]^2}} \qquad (1)$$

where:

A = vibration amplitude
Fo = exciting force
K = spring rate
w = period of vibration
n = damping factor
n_c = critical damping factor
m = mass of vibrating object
p = K/m

In equation (1), the term Fo/K represents the deflection of the system under a statically-applied load Fo. The expression

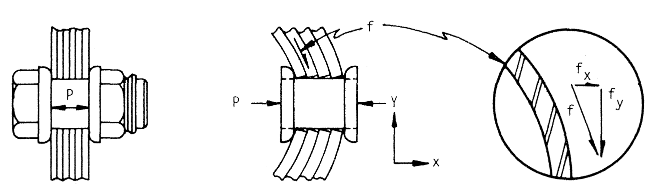

Figure 7. Frictional Damping in Metal-Disc Coupling.

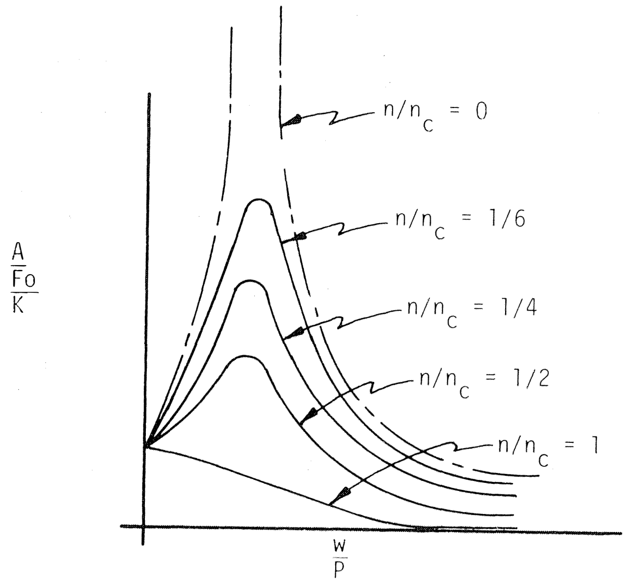

Figure 8. Resonant Vibration Amplitude With Damping.

on the right-hand side then represents the dynamic amplification, or magnification factor, and relates the static and dynamic deflections. The relationship of these parameters for various ratios of frequency (w/p) and damping ratios (n/n_c) is shown on Figure 8. It should be noted that equation (1) is strictly valid only in the case of viscous damping. It has been demonstrated by Den Hartog (3) that this equation gives a very close approximation for frictional damping near resonance.

Of particular interest is the bahavior of the system at or near resonance; that is, where the frequency ratio w/p approaches unity. For small damping, maximum vibration amplitude occurs very near to w/p = 1.0. Substituting in (1), we have:

$$A_{RES} = \frac{\frac{Fo}{K}}{2\frac{n}{n_c}} \qquad (2)$$

where:

A_{RES} = vibration amplitude at resonance

It can be seen from this equation that even a small amount of frictional damping can be highly beneficial in restricting vibration amplitude at resonance.

Calculation of resonant frequency where coulomb damping exists is a relatively simple matter, and the equation is identical to that for undamped free vibration (4). Derivations are contained in any textbook on mechanics and may be expressed as follows:

$$P_{cr} = \frac{1}{2\pi}\sqrt{\frac{K}{m}} \qquad (3)$$

where:

P_{cr} = natural, or resonant frequency or hz

A practical application of the above is to determine the resonant frequency of a coupling center member in a drive system when subjected to a once-per-revolution excitation. An equation, analagous to (3) above is given (5) by:

$$f_n = \frac{60}{2\pi}\sqrt{\frac{1536\,K_{EFF}}{W}} \qquad (4)$$

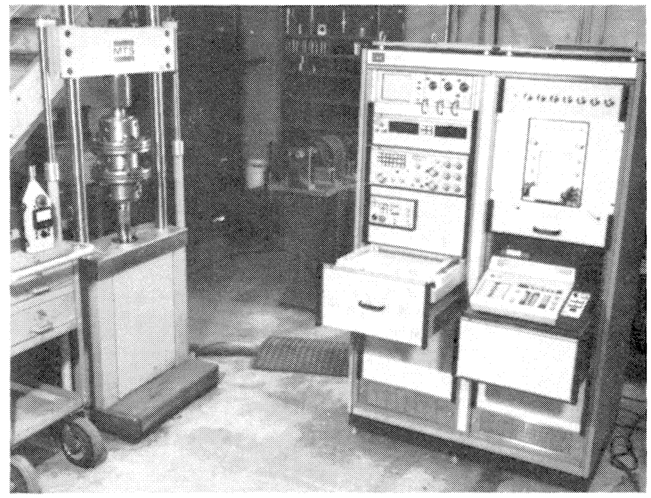

Figure 9. MTS Servo-Hydraulic Testing Machine.

where:

f_n = resonant frequency of center member - cyc./min.
K_{EFF} = effective (published) spring rate - lb./in.
W = center member weight - lb.

EXPERIMENTAL PROGRAM

Description of Testing Apparatus

The Corporate Research and Development Department of Rexnord Inc. is located in Milwaukee, Wisconsin and performs research, development, and testing services for all divisions of the Corporation. In 1975, an MTS servo-hydraulic testing machine was installed in this facility (see Figure 9). The system consists of 1) a hydraulic power supply, 2) a servo valve with maximum response rate of 250hz, 3) a loading frame, and 4) a computerized control center. The control center permits the input of a preprogrammed loading pattern of up to 2,000 discrete data points and can also be used to generate a sinusoidal motion.

Measurement of the vibration characteristics of laminated-disc flexible couplings is accomplished by mounting a double-flexing coupling in a vertical position in the main load frame. The mounting arrangement is shown in Figure 10. The upper hub is attached rigidly to a load cell by means of a locking pin and spanner nut; the hub must be tightly clamped, without backlash, to avoid introducing extraneous motions into the test setup. The lower hub is attached in a similar manner to a hydraulic piston which is actuated by the servo valve.

Instrumentation is provided to measure dynamic deflection of the lower (input) hub and coupling center member. The force transmitted through the coupling to the upper hub is also recorded. The frequency of excitation is controlled and monitored on the control terminal. It is also possible to determine phase relationship between the lower hub and center member from traces on either an oscilloscope or an X-Y recorder.

Test Procedure

After verifying that the coupling is correctly installed and all bolts are properly tightened, the first step is to generate a "static" force versus deflection curve, such as shown on Figure 11. Using this data, and knowing the center member weight, it is possible to calculate the undamped natural frequency of the system. This is extremely important

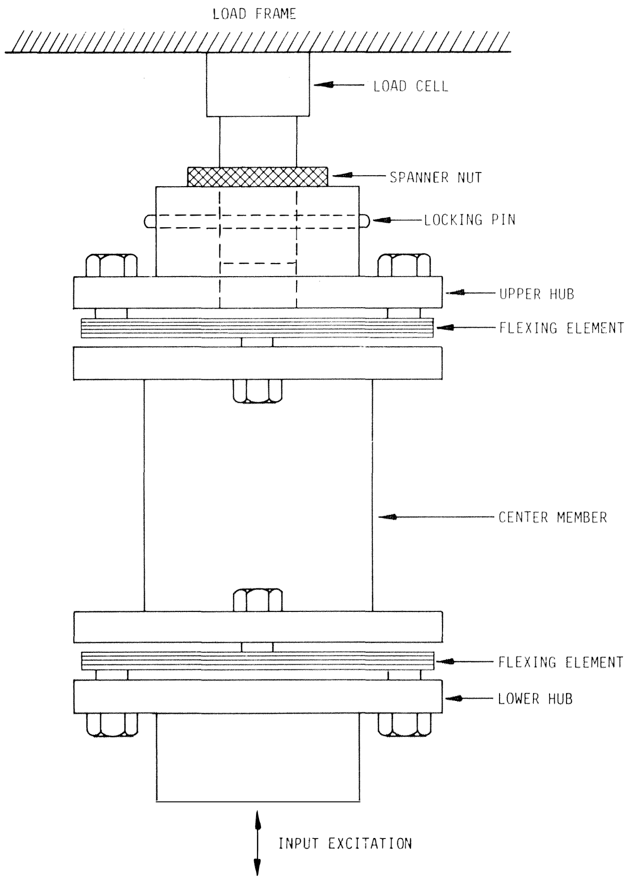

Figure 10. Experimental Coupling Mounting Arrangement.

since it provides a means of correlating analytical values with test results.

The procedure for dynamic analysis is as follows: With the coupling in its "neutral" position, the lower hub is oscillated at slowly increasing frequencies while at the same time monitoring center member displacement. The center member displacement remains relatively constant at approximately one-half the input displacement until the resonsant frequency is approached, at which time its amplitude increases rapidly. The maximum center member displacement, the frequency at which it occurs, and the force transmitted to the upper hub are all noted.

The entire procedure is repeated for various incremental values of coupling pre-compression and pre-extension. Since the coupling spring rate characteristics are non-linear, it is expected that the frequency response will change.

Test Results

Test data shown in this report is based on evaluation of a "Thomas"* size 450 Series 52 coupling with tubular center spacer and stainless steel flexing elements.

1. Static Testing

The static force-deflection and spring rate data are shown on Figure 11. Using the formula given by equation (4), the resonant frequency can be derived. Note that here the data is expressed in cycles per minute to facilitate correlation with machinery-produced excitations which are normally expressed as "once-per-revolution," or in a similar manner.

*Registered TM, Rexnord Inc., Milwaukee, Wisconsin

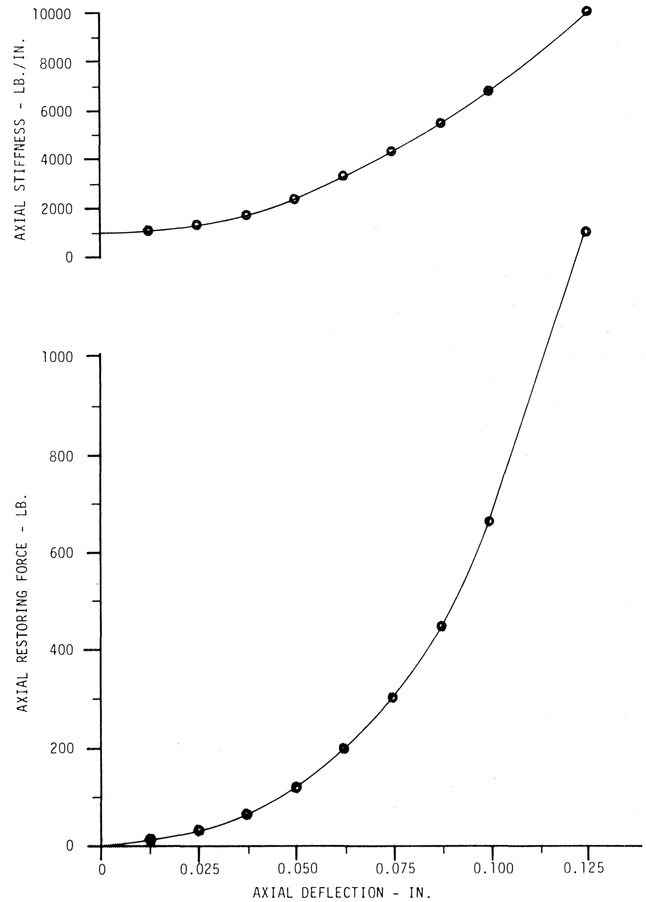

Figure 11. Measured Static Spring Rate Data

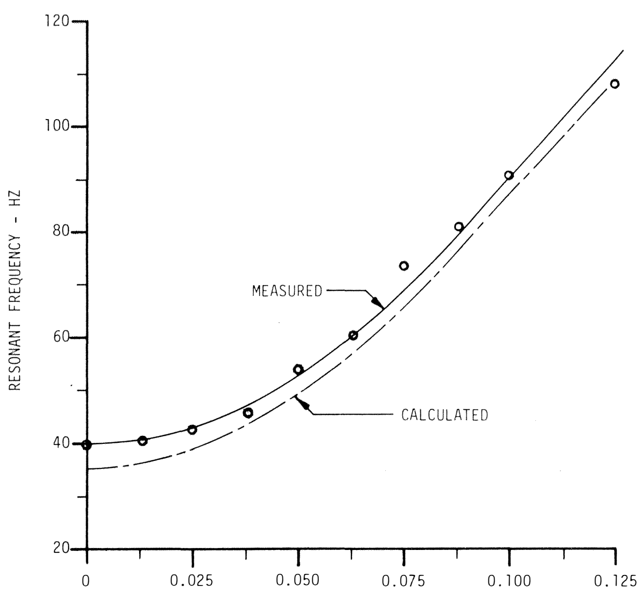

Figure 12. Comparison of Measured and Calculated Resonant Frequency.

2. Measured Resonant Frequency

The measured resonant frequency of the test coupling is shown on Figure 12 for various positions of extension and compression. Note that no difference between the two modes is discernable. As expected, the resonant frequency increases as the amount of preload increases. The experimental and calculated values are compared, and the experimental values are seen to be just slightly higher than calculated. The only notable divergence is in the area near neutral which is the so-called "free end float" range, characteristic of this type coupling.

3. Vibration Amplitude at Resonance — Magnification Factor

Coupling center member vibration amplitude increases with exciting frequency as seen from Figure 13. On this curve, the magnification factor, or the ratio of output to input vibration amplitude, is shown as a function of excitation frequency. It appears that frictional damping has less effect at the higher forcing frequencies resulting in a greater resonant vibration amplitude. Note that tests were run over a range of input vibration amplitudes — 0.020 inch peak-to-peak at lower frequencies down to 0.0025 peak-to-peak at the upper range.

4. Response at Resonant Frequency

Typical response curves for the coupling tested are shown on Figure 14 for various values of coupling pre-extention (e.g., at various spring rates). This type of coupling exhibits a rather narrow response, primarily at the higher frequencies. The non-linear spring rate of the coupling is thought to be a factor since the vibration amplitude will change the spring rate sufficiently to shift the coupling out of resonance.

5. Determination of Damping Characteristics

From the standpoint of practical application, the effect of damping on resonant vibration amplitude is of particular interest. Referring to equation (2), we can see that the critical damping ratio, n/n_c, is related to the magnification factor which can be defined as

$$\frac{A_{RES}}{\frac{Fo}{K}}$$

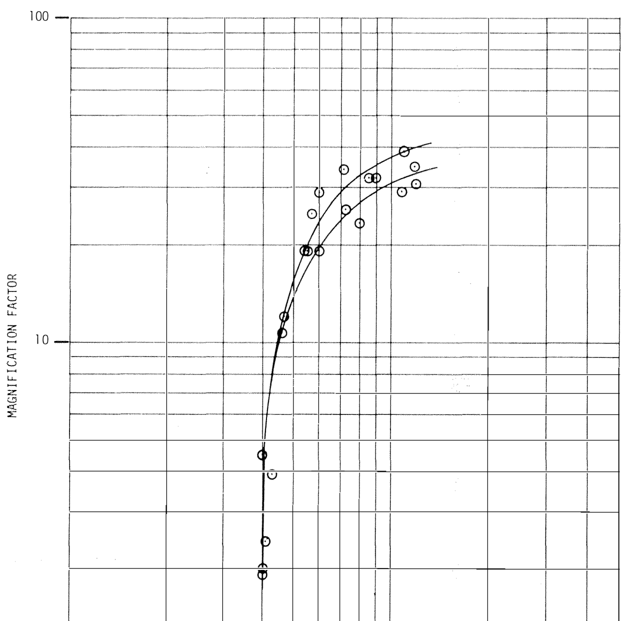

Figure 13. Magnification Factor At Resonance.

(Note that the expression in the denominator is the deflection of a spring constant K when subjected to a static force Fo). An approximate correlation between the critical damping ratio and resonant frequency was found to exist (Figure 15). Although there is some scatter, there is a definite indication that a lower limit is being approached in the range of $n/n_c = 0.015$. Using this lower limit

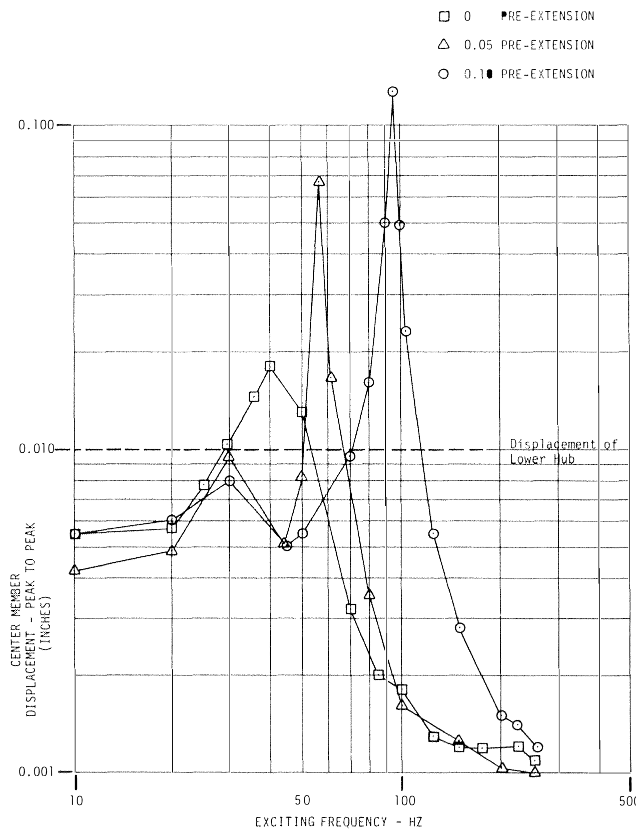

Figure 14. Response of Metal-Disc Coupling.

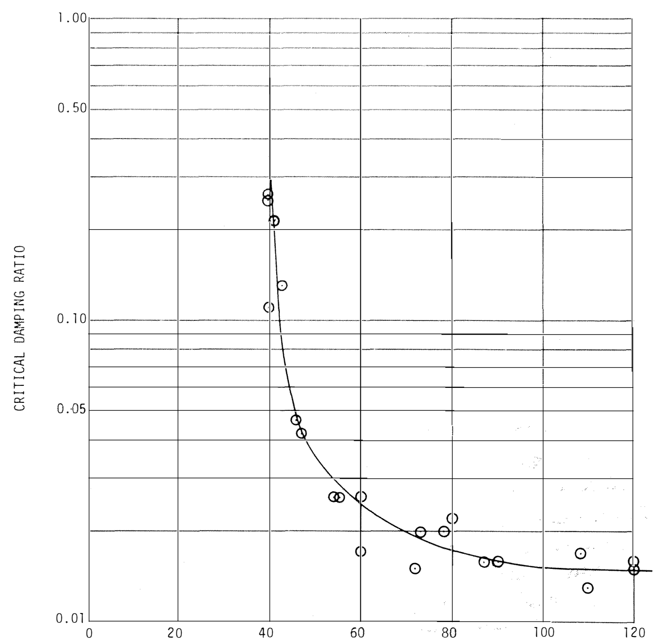

Figure 15. Measured Critical Damping Ratio.

will allow an approximation of resonant vibration amplitude for the conditions tested. Since the damping is primarily friction-related, considerable scatter can be expected due to variations in friction coefficient, localized surface asperities on adjacent laminations, etc. Further testing is required to more closely define this characteristic.

CONCLUSIONS

1. A theoretical analysis of a drive system incorporating a metal-flexing coupling will show that the potential exists for a resonant vibration of the coupling center member. Results of the testing program confirm that such a condition exists providing that an external exciting force is present. It is possible to measure and predict with reasonable accuracy the response of a metal-disc type flexible coupling to such an excitation.

2. Axial spring rate data taken from static tests can be used to closely estimate the resonant frequency of a metal-flexing, disc type coupling based on results of testing. Measured values agreed with calculated figures within ± 3.3%.

3. The presence of damping in laminated disc couplings is sufficient to prevent resonant vibration of the coupling unless an external exciting force is present.

4. Vibration amplitude of the coupling center member can be as much as 30-35 times the magnitude of the input vibration when the coupling is excited at its resonant frequency. Based on test results, the magnification factor (ratio of output to input vibration amplitude) varied directly with resonant frequency and was independent of input amplitude.

5. Frictional damping, resulting from microscopic motion between adjacent laminations in a disc type coupling, appears to have a significant effect on resonant vibration amplitude. Critical damping ratios ranging between approximately 0.015 and 0.250 were found to exist on the test unit.

6. The non-linear spring rate exhibited by the test coupling served to limit the response of the coupling to frequencies at or near the resonant point, as evidenced by rapid increase, and subsequent rapid decrease in vibration amplitude when passing through resonance.

7. Field experience by manufacturers and users of turbomachinery has shown that resonant axial vibration of a metal-flexing coupling can at times cause problems which are reflected through the entire drive train. In the case of laminated-disc couplings, the problem rises only when an external forcing function exists. This could be a result of aerodynamic or hydraulic fluctuations in the machine train, out-of-square thrust collars, gearing inaccuracies, or electrical excitations (in the case of motor-driven equipment). It is usually possible to avoid operating the coupling at or near resonance if the condition is anticipated during the system design stage. However, such problems do not always arise until after a machine is in service. More information is needed on the nature and magnitude of external excitations.

8. Since the nature and magnitude of axially-directed exciting forces are not always known during the design phase of a drive system, it is desirable to have the means of "tuning" the coupling after installation in the field. The most expedient means is generally to change the coupling stiffness. The laminated-disc type coupling offers this ability to change stiffness without the cost and downtime attendant with changing major components of the coupling.

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